MTH241 Fall 2024: Quiz 01

Name: One piece
UID:

Closed book, no calculator, show your work clearly.

1. (5pt) Show that $x^2+y^2+z^2+2x-4y+6z-8=0$ is the equation of a sphere. Find the center and radius of this sphere. (Grading: **1pt**: working; **2pt**: proof; **2pt**: center and radius)

$(x^2+2x+1)-1+(y^2-4y+4)-4+(z^2+6z+9)-9-8=0$

$\Rightarrow (x+1)^2+(y-2)^2+(z+3)^2=22$

center $=(-1,2,-3)$

radius $=\sqrt{22}$

2. (5pt) Let $\vec{a}=\vec{i}+2\vec{j}+3\vec{k}$, and $\vec{b}=4\vec{i}+5\vec{j}+6\vec{k}$. Find the projection of $\vec{a}$ onto $\vec{b}$ and the projection of $\vec{b}$ onto $\vec{a}$. (Grading: **1pt**: working; **2pt**: first projection; **2pt**: second projection)

- $a=(1,2,3) \rightsquigarrow \|a\|=\sqrt{1+4+9}=\sqrt{14}$
  $b=(4,5,6) \rightsquigarrow \|b\|=\sqrt{16+25+36}=\sqrt{77}$

$a\cdot b=4+10+18=32$

- $\text{pr}_b a=\frac{a\cdot b}{\|b\|^2}b=\frac{32}{77}b$
- $\text{pr}_a b=\frac{a\cdot b}{\|a\|^2}a=\frac{32}{14}a=\frac{16}{7}a$